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Peristaltic Transport of a Couple Stress Fluid With Slip Effects in an Inclined Channel

Monayya Mareppa[a] and N. G. Sridhar[b]

[a]Department of Mathematics, Govt. Degree College,Yadgir-585 202.Karnataka, India

[b]Department of Mathematics, Govt. First Grade College, Sedam-585 222, Karnataka, India

Abstract - The present paper investigates the peristaltic transport of a couple stress fluid in a two dimensional inclined channel with effect of slip parameter. The effects of various physical parameters on velocity, pressure gradient and friction force have been discussed & computed numerically. The effects of various key parameters are discussed with the help of graphs.

Keywords: *Peristaltic flow, Couple stress fluid, slip parameter, inclined channel.*

1. INTRODUCTION

Peristalsis is now well known to physiologists to be one of the major mechanisms for fluid transport in many biological systems. In particular, a mechanism may be involved in swallowing food through the esophagus, in urine transport form the kidney to the bladder through the urethra, in movement of chyme in the gastro –intestinal tract, in the transport of spermatozoa in the ductus efferent of the male reproductive tracts and in the cervical canal, in movement of ovum in the female fallopian tubes, in the transport of lymph in the lymphatic vessels, and in the vasomotion of small blood vessel such as arterioles, venules and capillaries.

Srivastava et.al., [1] peristaltic transport of a physiological fluid: part I flow in non- uniform geometry. Latham [2] investigated the fluid mechanics of peristaltic pump and science. Ramchandra and Usha [3] studied the influence of an eccentrically inserted catheter on the peristaltic pumping in a tube under long wavelength and low Reynolds numbers approximations. Gupta and Sheshadri [4] studied peristaltic transport of a Newtonian fluid in non-uniform geometries. Srivastava and Srivastava [5] have investigated the effect of power law fluid in uniform and non-uniform tube and channel under zero Reynolds number and long wavelength approximation. Rathod and Asha [6] have investigated peristaltic transport of a couple stress fluid In a uniform and non-uniform annulus. Habtu and

Radhakrishnamacharya [7] studied dispersion of a solute in peristaltic motion of a couple stress fluid through a porous medium with slip condition. Rathod et.al., [8] have investigated peristaltic pumping of couple stress fluid through non - erodible porous lining tube wall with thickness of porous material. Mekhemier and Abd elmaboud [9] peristaltic flow of a couple stress fluid in an annulus: Application of an endoscope. Rathod and Sridhar [10] investigated peristaltic transport of couple stress fluid in uniform and non-uniform annulus through porous medium. Rathod and Asha [11] studied effect of couple stress fluid and an endoscope in peristaltic motion. Alsaedi et.al., [12] studied peristaltic flow of couple stress fluid through uniform porous medium. Rathod and Pallavi [13] studied the influence of wall properties on MHD peristaltic transport of dusty fluid. Rathod and Mahadev [14] studied of ureteral peristalsis with Jeffrey fluid flow. Rathod and Mahadev [15] studied slip effects and heat transfer on MHD peristaltic flow of Jeffrey fluid in an inclined channel. Rathod and Laxmi [16] studied peristaltic transport of a conducting fluid in an asymmetric vertical channel with heat and mass transfer. Rathod and Sridhar [17] investigated Peristaltic flow of a couple stress fluid in an inclined channel. Rathod et. al., [18] studied peristaltic transport of a conducting couple stress fluid through a porous medium in a channel.

The present research aimed is to investigate the interaction of peristalsis for the flow of a couple stress fluid with effect of slip parameter in a two dimensional inclined channel. The computational analysis has been carried out for drawing velocity profiles, pressure gradient and frictional force.

II. Formulation of the problem:

We consider a peristaltic flow of a Couple stress fluids through two-dimensional channel of width 2a and inclined at an angle α to the horizontal. Symmetric with respect to its axis. The walls of the channel are assumed to be flexible.

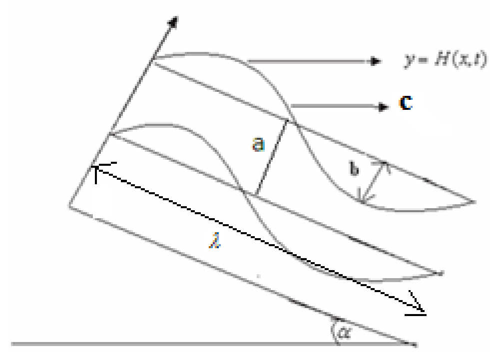

Fig.1. Schematic diagram of the inclined channel.

The wall deformation is given by

$$H(x,t) = a + bCos(\frac{2\pi}{\lambda}(X - ct)) \qquad (1)$$

Where ‘b’ is the amplitude of the peristaltic wave, ‘c’ is the wave velocity, ‘λ’ is the wave length, t is the time and X is the direction of wave propagation.

Neglecting the body force and the body couples, the continuity equation and equations of motion (Mekheimer 2002) through a porous medium are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (2)$$

Navier Stokes equations are:

$$\rho(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y})=-\frac{\partial p}{\partial x}+\mu\nabla^2 u-\eta^*\nabla^4 u+\rho g\sin\alpha \quad (3)$$

$$\rho(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y})=-\frac{\partial p}{\partial y}+\mu\nabla^2 v-\eta^*\nabla^4 v-\rho g\cos\alpha \quad (4)$$

Where, $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}, \nabla^4=\nabla^2\nabla^2$

u and v are velocity components, 'p' is the fluid pressure, 'ρ' is the density of the fluid, 'μ' is the coefficient of viscosity, 'η^*' is the coefficient of couple stress, 'g' is the gravity due to acceleration and 'α' angle of inclination.

Introducing a wave frame (x, y) moving with velocity c away from the fixed frame (X, Y) by the transformation

$$x=X-ct, y=Y, u=U, v=V, p=P(X,t) \quad (5)$$

We introduce the non-dimensional variables:

$$x^*=\frac{x}{\lambda},\ y^*=\frac{y}{a}, u^*=\frac{u}{c}, v^*=\frac{v}{c\delta},\ t^*=\frac{tc}{\lambda}, (\eta^*)^*=\frac{\eta^*}{a}, p^*=\frac{pa^2}{\mu c\lambda}, G=\frac{\rho g a^2}{\mu c}, \phi=\frac{b}{a}, h=\frac{H}{a} \quad (6)$$

Equation of motion and boundary conditions in dimensionless form becomes

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad (7)$$

$$\operatorname{Re}\delta(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y})=-\frac{\partial p}{\partial x}+(\delta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2})-\frac{1}{\gamma^2}(\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2})(\delta^2\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2})-G\sin\alpha \quad (8)$$

$$\operatorname{Re}\delta^3(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y})=-\frac{\partial p}{\partial y}+\delta^2(\delta^2\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2})-\frac{1}{\gamma^2}\delta^2(\delta^2\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2})(\delta^2\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2})-\rho g\delta\cos\alpha \quad (9)$$

Where, $\gamma^2=\frac{\eta^*}{\mu a^2}$ couple-stress parameter.

The dimensionless boundary conditions are:

$$\frac{\partial u}{\partial y}=0; \quad \frac{\partial^2 u}{\partial y^2}=0 \quad at \quad y=0$$

$$u=-k_n\frac{\partial u}{\partial y};\frac{\partial^2 u}{\partial y^2} \quad finite \quad at \quad y=\pm h=1+\phi Cos[2\pi x] \tag{10}$$

where, k_n is Knudsen number (slip parameter)

Using long wavelength approximation and neglecting the wave number δ, one can reduce Navier Stokes equations:

$$\frac{\partial p}{\partial y}=0 \tag{11}$$

$$\frac{\partial p}{\partial x}=\frac{\partial^2 u}{\partial y^2}-\frac{1}{\gamma^2}\frac{\partial^4 u}{\partial y^4}-G\sin\alpha \tag{12}$$

Solving the Eq.(12) with the boundary conditions (10), we get

$$u=\frac{\partial p}{\partial x}D(1-\frac{D}{\gamma^2})-G\sin\alpha * D \tag{13}$$

Where $D=\frac{y^2}{2}-\frac{h^2}{2}-\beta y$

The volumetric flow rate in the wave frame is defined by

$$q=\int_0^h u dy=\frac{\partial p}{\partial x}(A-\frac{1}{\gamma^2}(\frac{2h^5}{15}+\frac{h^4\beta}{4}+\frac{h^3\beta^2}{3}))-GA\sin\alpha \tag{14}$$

Where, $A=\frac{h^3}{6}-\frac{h^2\beta}{2}-\frac{h^3}{2}$

The expression for pressure gradient from Eq.(14) is given by

$$\frac{\partial p}{\partial x}=\frac{q+GA\sin\alpha}{(A-\frac{1}{\gamma^2}(\frac{2h^5}{15}+\frac{h^4\beta}{4}+\frac{h^3\beta^2}{3}))} \tag{15}$$

The instantaneous flux Q (x, t) in the laboratory frame is

$$Q(x,t)=\int_0^h (u+1)dy = q+h \tag{16}$$

The average flux over one period of peristaltic wane is $\overline{Q}$

$$\overline{Q}=\frac{1}{T}\int_0^T Qdt = q+1 \tag{17}$$

From equations (15) and (17), the pressure gradient is obtained as

$$\frac{\partial p}{\partial x}=\frac{(\overline{Q}-1)+GA\sin\alpha}{(A-\frac{1}{\gamma^2}(\frac{2h^5}{15}+\frac{h^4\beta}{4}+\frac{h^3\beta^2}{3}))} \tag{18}$$

The pressure rise (drop) over one cycle of the wave can be obtained as

$$\Delta P=\int_0^1 (\frac{dp}{dx})dx \tag{19}$$

The dimensionless frictional force F at the wall across one wavelength is given by

$$F=\int_0^1 h(-\frac{dp}{dx})dx \tag{20}$$

III. RESULT AND DISCUSSIONS

In this section we have presented the graphical results of the solutions axial velocity u, pressure rise ΔP, friction force F for the different values of couple stress (γ), gravitational parameter (G), angle of inclination (α), slip parameter (β) and amplitude (ϕ).

The axial velocity is shown in **Figs. (2 to 5**). The Variation of u with γ, we find that u depreciates with increase in γ (**Fig. 2**). The Variation of u with gravitational parameter G shows that for u increases with increasing in G (**Fig 3**). The Variation of u with angle of inclination α shows that for u increases with increasing in α (**Fig 4**). The axial velocity u is exhibit in (**fig. 5**) for different values of slip parameter β. It is found that the velocity u is increases with increasing β.

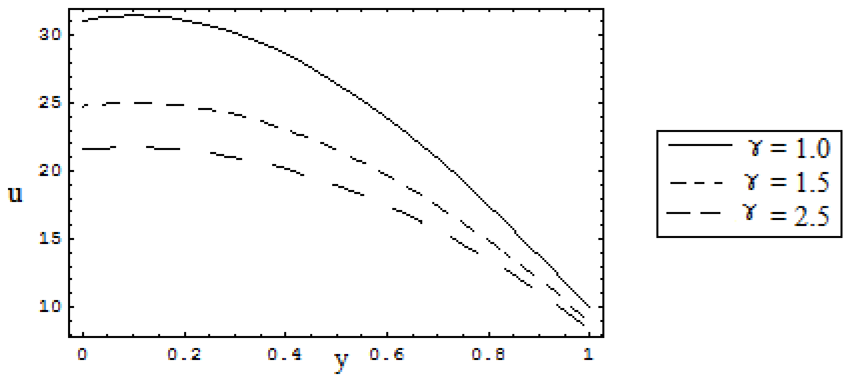

Fig. 2: Effect of γ on u, when β □ □ □ □ x□ □ □ □ □p=-25,G = 6, □ = □ /4& ϕ = 0.2

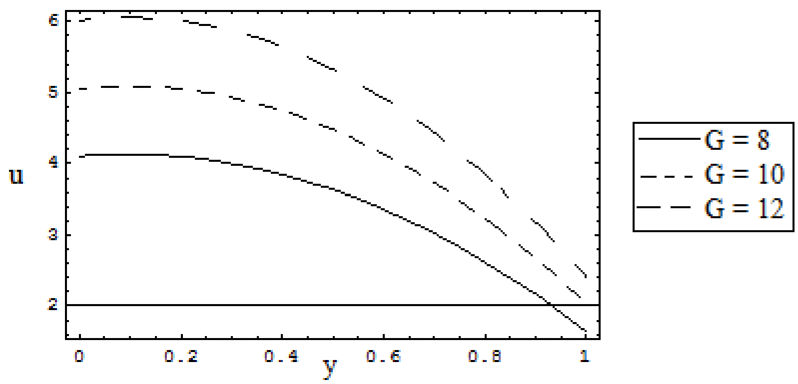

Fig. 3: Effect of **G** on u□ when γ □ □□ β □ □ □ □ x□ □ □ □ □p=-.25,□ = □ /4&□ ϕ = 0.2

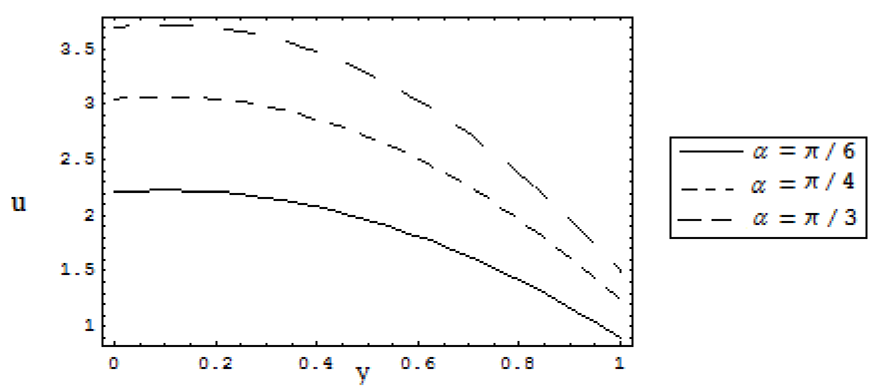

Fig. 4: Effect of α on u□ when γ □ □□ β □ □ □ □ x□ □ □ □p=-.25,□ □ =0.2& G = 6□

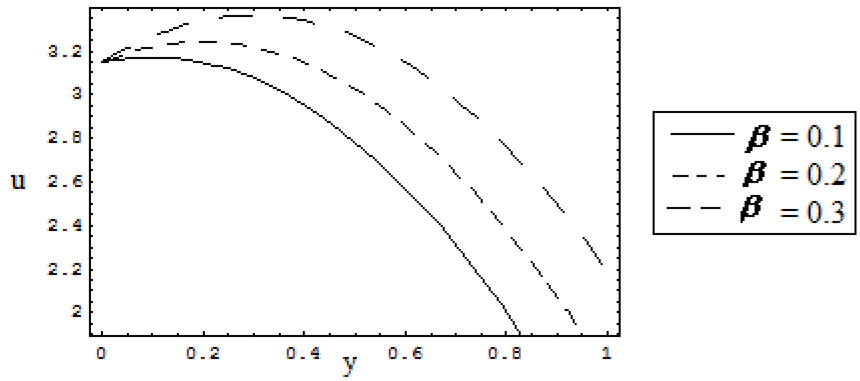

Fig. 5: Effect of β on u□ □when γ □ □ □□ x□ □ □ □□ □p=-.25, G = 6, □ =0.2& □ = □ /4.

The variation of pressure rise ΔP is shown in **Figs. (6 to 9)** for a different values of γ, G, α & β. We find that ΔP increases with increase in γ (**Fig. 6**). The Variation of ΔP with gravitational parameter G shows that for ΔP increases with increasing in G (**Fig 7**). The Variation of ΔP with angle of inclination α shows that for ΔP increases with increasing in α (**Fig 8**). The axial velocity ΔP is exhibit in (**fig. 9**) for a different values slip parameter β. It is found that the velocity ΔP is decreases with increasing β.

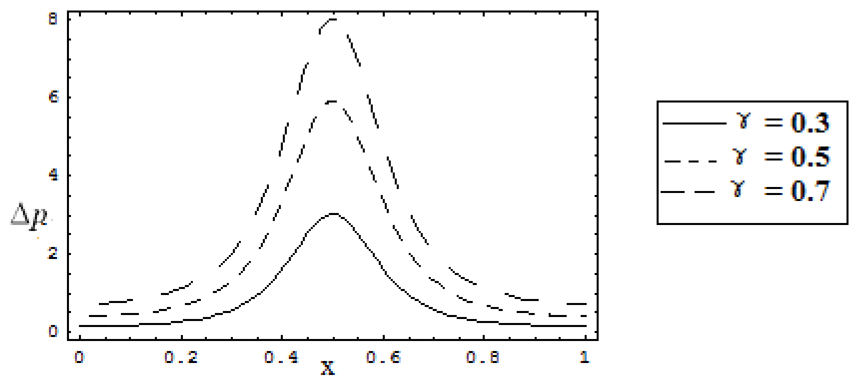

Fig. 6: Effect of γ on Δp □ when β □ □□ G = 4, □ = □ /4, Q = 0& □ =0.6□

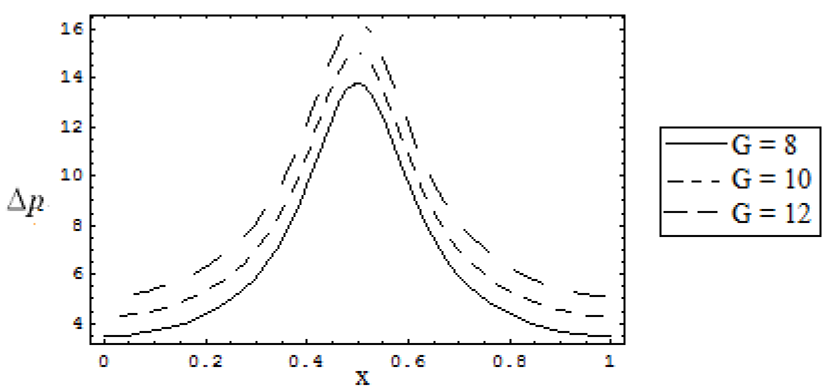

Fig. 7: Effect of **G** on Δp □ when γ □ □ □ □□ β □ □□ □□ = □ /4, □ □ =0.6& Q = 0.□

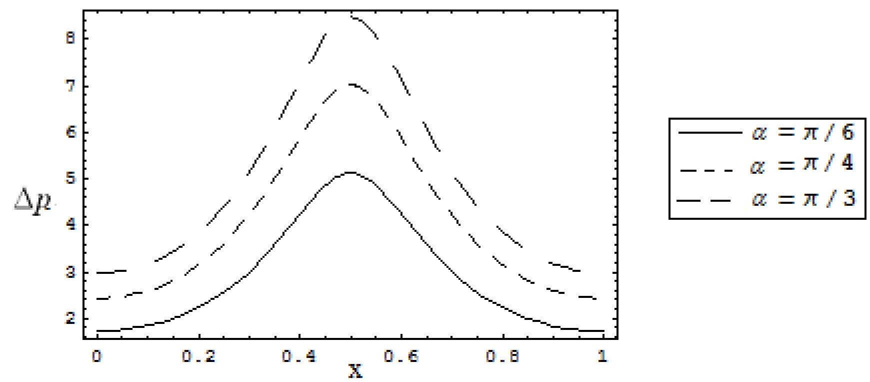

Fig. 8: Effect of α on Δp □ when γ □ □ □ □ β □ □ □ □Q=0,□ □ =0.6□ □G = 20.

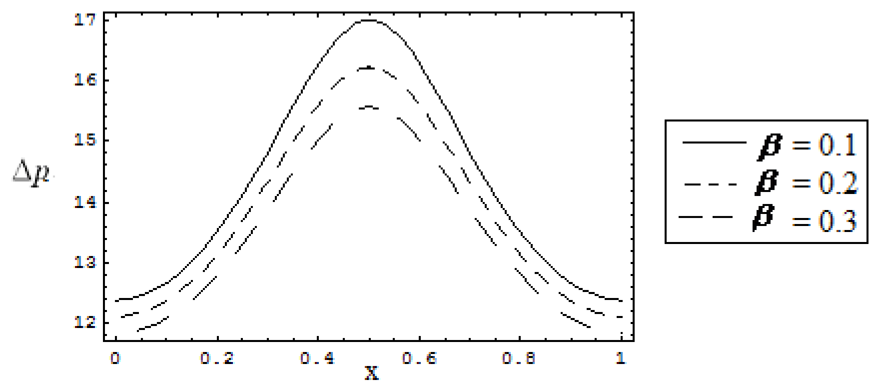

Fig. 9: Effect of β on Δp □ when γ □ □□ □□G = 20, □ = □ /4,□ =0.2& Q=0.

The variation of friction force F is shown in **Figs. (10 to 13)** for a different values of γ, G, α & β. We find that F depreciates with increase in γ (**Fig. 10**). The Variation of F with gravitational parameter G shows that for F decreases with increasing in G (**Fig. 11**). The Variation of F with angle of inclination α shows that for F decreases with increasing in α (**Fig 12**). The friction force F is exhibit in (**fig. 13**) for different values of slip parameter β. It is found that the F is increases with increasing β.

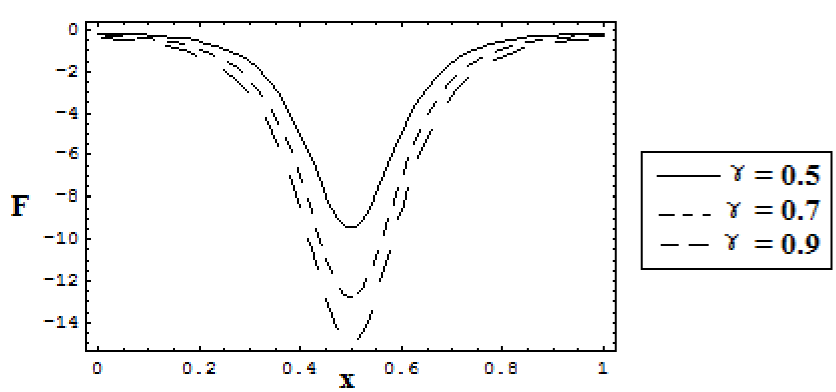

Fig. 10: Effect of γ on F□ whenG = 4□ β □□□□ = □ /4,Q = 0 □□ □ □□ =0.6

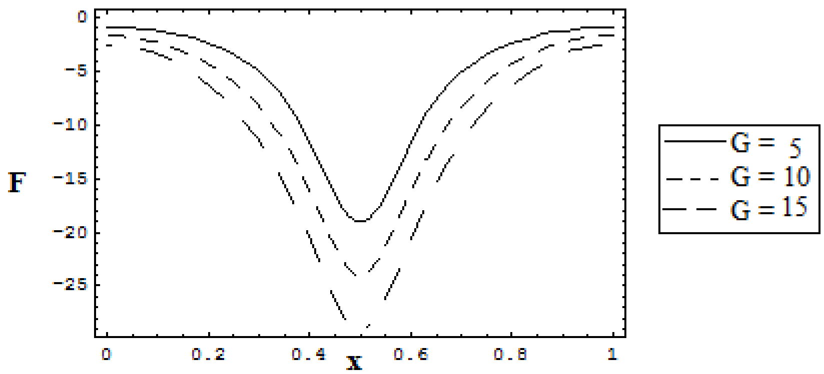

Fig. 11: Effect of **G** on F□ □ when γ □ □ □ □ β □□□□ = □ /4, Q = 0□ □ □□ =0.6

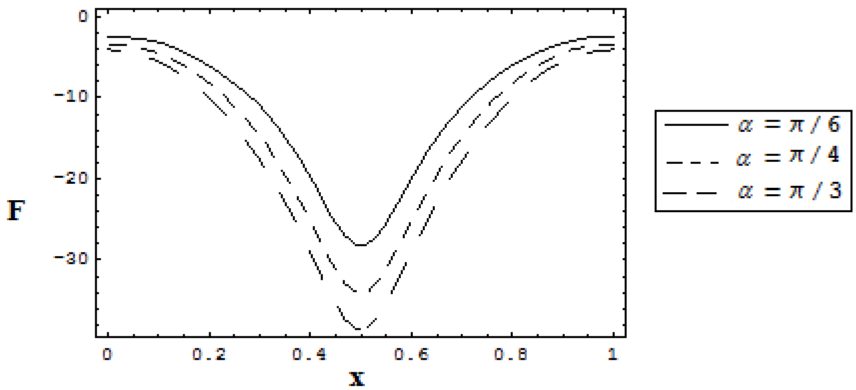

Fig. 12: Effect of α on F□ □ when γ □□ □ □□ β □ □□ G = 20, Q=0 □ □□ =0.6

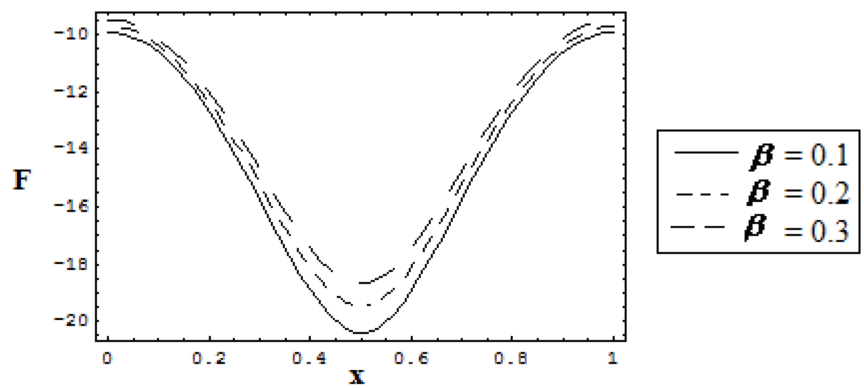

Fig. 13: Effect of β on F□ □ when γ □ □ □ □ G = 20, □ = □ /4, Q=0& □ =0.2

IV. CONCLUSION

In this paper we presented a theoretical approach to study the peristaltic flow of a couple stress fluid in an inclined channel with effect of slip parameter. The governing Equations of motion are solved analytically. Furthermore, the effect of various values of parameters on Velocity, Pressure rise and Friction force have been computed numerically and explained graphically. We conclude the following observations:

1. The velocity u increases with increase in gravitational parameter G, angle of inclination α & slip parameter β but, decreases with increase in couple stress parameter γ.

2. Pressure rise ΔP decreases with increase in slip parameter β but, increases with increase in gravitational parameter G, angle of inclination α & couple stress parameter γ.

3. The friction force F has increases with increase in slip parameter β but, decreases with increase in gravitational parameter G, angle of inclination α & couple stress parameter γ.

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