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Peristaltic Flow of A Couple Stress Fluids in an Inclined Channel

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Abstract - The present paper investigates the peristaltic motion of a couple stress fluid in a two dimensional inclined channel. The effects of various physical parameters on velocity, pressure gradient and friction force have been discussed & computed numerically. The effects of various key parameters are discussed with the help of graphs.

Keywords: *Peristaltic flow, Couple stress fluid, inclined channel.*

1. INTRODUCTION

Peristalsis is known to be one of the main mechanisms of transport for many physiological fluids, which is achieved by the passage of progressive waves of area contraction and expansion over flexible walls of a tube containing fluid. This mechanism is found in many physiological situations like urine transport from kidney to the bladder through the ureter, swallowing food through the esophagus, movement of chyme in the gastrointestinal tract, transport of spermatozoa in the ducts efferent's of the male reproductive organ, movement of ovum in the female fallopian tube, vasomotion of small blood vessels, motion of spermatozoa in cervical canal, transport of bile in bile duct. Some worms like earth-worm use peristalsis for their locomotion. Some biomedical instruments such as heart-lung machine work on this principle. Mechanical devices like finger pumps, roller pumps use peristalsis to pump blood, slurries and corrosive fluids. It is also speculated that peristalsis may be involved in the translocation of water in tall trees. The translocation of water involves its motion through the porous matrix of the trees. The peristaltic transport of a toxic liquid is used in nuclear industry so as not to contaminate the outside environment. Various studies on peristaltic transport, experimental as well as theoretical, have been carried out by many researchers to explain peristaltic pumping in physiological systems. Srivastava et.al., [1] peristaltic transport of a physiological fluid: part I flow in non- uniform geometry. Latham [2] investigated the fluid mechanics of peristaltic pump and science. Ramchandra and Usha [3] studied the influence of an eccentrically inserted catheter on the peristaltic pumping in a tube under long wavelength and low Reynolds numbers approximations. Gupta and Sheshadri [4] studied peristaltic transport of a Newtonian fluid in non-uniform geometries. Srivastava and Srivastava

[7] have investigated the effect of power law fluid in uniform and non-uniform tube and channel under zero Reynolds number and long wavelength approximation. Rathod and Pallavi [23] studied the influence of wall properties on MHD peristaltic transport of dusty fluid. Rathod and Mahadev [26] studied of ureteral peristalsis with Jeffrey fluid flow. Rathod and Mahadev [27] studied slip effects and heat transfer on MHD peristaltic flow of Jeffrey fluid in an inclined channel. Rathod and Laxmi [29] studied peristaltic transport of a conducting fluid in an asymmetric vertical channel with heat and mass transfer.

The study of couple stress fluid is very useful in understanding various physical problems because it possesses the mechanism to describe rheological complex fluids such as liquid crystals and human blood. By couple stress fluid, we mean a fluid whose particles sizes are taken into account, a special case of non-Newtonian fluids. In further investigation many authors have used one of the simplification is that they have assumed blood to be a suspension of spherical rigid particles (red cells), this suspension of spherical rigid particles will give rise to couple stresses in a fluid. The theory of couple stress was first developed by Stokes in the year 1966 and represents the simplest generalization of classical theory which allows for polar effects such as presence of couple stress and body couples. A number of studies containing couple stress have been investigated by Rathod and Asha [8] have investigated peristaltic transport of a couple stress fluid In a uniform and non-uniform annulus. Habtu and Radhakrishnamacharya [12] studied dispersion of a solute in peristaltic motion of a couple stress fluid through a porous medium with slip condition. Rathod et.al., [14] have investigated peristaltic pumping of couple stress fluid through non - erodible porous lining tube wall with thickness of porous material. Mekhemier and Abd elmaboud [15] peristaltic flow of a couple stress fluid in an annulus: Application of an endoscope. Alsaedi, Ali et.al., [20] studied peristaltic flow of couple stress fluid through uniform porous medium. Rathod and Sridhar [16] investigated peristaltic transport of couple stress fluid in uniform and non-uniform annulus through porous medium. Rathod and Asha [17] studied effect of couple stress fluid and an endoscope in peristaltic motion.

Flow through porous media has been of considerable interest in the recent years due to the potential application in all fields of Engineering, Geo-fluid dynamics and Biomechanics. For example study of flow through porous media is immense use to understand transport process in lungs, in kidneys, gallbladder with stones, movement of small blood vessels and tissues cartilage and bones etc. Most of the tissues in the body (e.g. bone, cartilage, muscle) are deformable porous media. The proper functioning of such materials depends crucially on the flow of blood, nutrients and so forth through them. Porous- medium models are used to understand various medical conditions (such as tumor growth) and treatments (such as injections). Raptis and Peridikis [9] are investigated flow of a viscous fluid through a porous medium bounded by a vertical surface. El-dabe and El-Mohandis [10] have studied magneto hydrodynamic flow of second order fluid through a porous medium on an inclined porous plane. Ayman and Sobh [11] investigated peristaltic transport of a magneto-newtonian fluid through a porous medium. Rathod and Mahadev [13] investigated effect of thickness of the porous material on the peristaltic pumping of a jeffry fluid with non - erodible porous lining wall. Abd elmaboud and Mekheimer [18] study non-linear peristaltic transport of a second-order fluid through a porous medium. Rathod and Asha [19] effects of magnetic field and an endoscope on peristaltic motion. Rathod and Mahadev [21] studied effect of magnetic field on ureteral peristalsis in cylindrical tube. Rathod and Mahadev [22] study of ureteral peristalsis in cylindrical tube through porous medium. Rathod and Pallavi [24] investigated the influence of wall properties on peristaltic transport of dusty fluid through porous medium. Rathod and Pallavi [25] studied the effect of slip condition and heat transfer on MHD peristaltic transport through a porous medium with complaint wall. Rathod and Laxmi [28] investigated slip effect on peristaltic transport of a conducting fluid through a porous medium in an asymmetric vertical channel by Adomian decomposition method. Rathod and Laxmi [30] investigated effects of heat transfer on the peristaltic MHD flow of a Bingham fluid through a porous medium in an inclined channel. Rathod and Laxmi [31] studied effects of heat transfer on the peristaltic MHD flow of a Bingham fluid through a porous medium in a channel. Raghunath Rao and Prasad Rao [5] studied peristaltic flow of a couple stress fluid through a porous medium in a channel at low Reynolds number. Mekhemier [6] studied non-linear peristaltic transport a porous medium in an inclined planar channel. Jayarami Reddy et.al., [32] studied peristaltic flow of a Williamson fluid in an inclined planar channel under the effect of a magnetic field. Subba Reddy et.al., [33] studied peristaltic transport of Williamson fluid in a channel under the effect of a magnetic field.

The present research aimed is to investigate the interaction of peristalsis for the flow of a couple stress fluid in a two dimensional inclined channel. The computational analysis has been carried out for drawing velocity profiles, pressure gradient and frictional force.

II. Formulation of the problem

We consider a peristaltic flow of a Couple stress fluids through two-dimensional channel of width 2a and inclined at an angle α to the horizontal. Symmetric, with respect to its axis. The walls of the channel are assumed to be flexible.

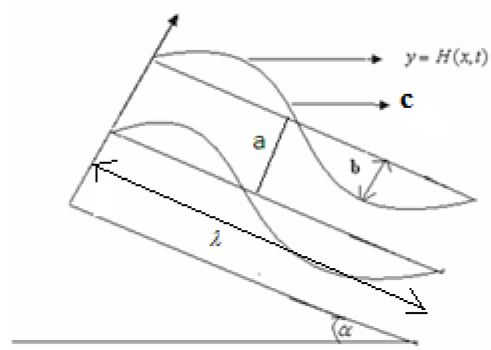

Fig.1.Schematic diagram of the inclined channel.

The wall deformation is given by

$$H(x,t) = a + bCos(\frac{2\pi}{\lambda}(X - ct)) \tag{1}$$

Where 'b' is the amplitude of the peristaltic wave, 'c' is the wave velocity, 'λ' is the wave length, t is the time and X is the direction of wave propagation.
Neglecting the body force and the body couples, the continuity equation and equations of motion (Mekheimer 2002) through a porous medium are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}$$

Navier Stokes equations are:

$$\rho(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}) = -\frac{\partial p}{\partial x} + \mu\nabla^2 u - \eta^*\nabla^4 u + \rho g \sin\alpha \tag{3}$$

$$\rho(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}) = -\frac{\partial p}{\partial y} + \mu\nabla^2 v - \eta^*\nabla^4 v - \rho g \cos\alpha \tag{4}$$

Where, $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \nabla^4 = \nabla^2 \nabla^2$

u and v are velocity components, 'p' is the fluid pressure, 'ρ' is the density of the fluid, 'μ' is the coefficient of viscosity, 'η^*' is the coefficient of couple stress, 'g' is the gravity due to acceleration and 'α' angle of inclination.
Introducing a wave frame (x, y) moving with velocity c away from the fixed frame (X, Y) by the transformation

$$x = X - ct, y = Y, u = U, v = V, p = P(X,t) \tag{5}$$

We introduce the non-dimensional variables:

$$x^* = \frac{x}{\lambda}, \; y^* = \frac{y}{a}, u^* = \frac{u}{c}, \; v^* = \frac{v}{c\delta}, \; t^* = \frac{tc}{\lambda}, \; (\eta^*)^* = \frac{\eta^*}{a}, \; p^* = \frac{pa^2}{\mu c \lambda}, G = \frac{\rho g a^2}{\mu c} \tag{6}$$

Equation of motion and boundary conditions in dimensionless form becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{7}$$

$$\operatorname{Re}\delta\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \left(\delta^2\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - \frac{1}{\gamma^2}\left(\delta^2\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left(\delta^2\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) - G\sin\alpha \tag{8}$$

$$\operatorname{Re}\delta^3\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \delta^2\left(\delta^2\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \frac{1}{\gamma^2}\delta^2\left(\delta^2\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\left(\delta^2\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \rho g\delta\cos\alpha \tag{9}$$

Where, $\gamma^2 = \frac{\eta^*}{\mu a^2}$ couple-stress parameter.

The dimensionless boundary conditions are:

$$\frac{\partial u}{\partial y} = 0; \quad \frac{\partial^2 u}{\partial y^2} = 0 \quad at \quad y = 0$$

$$u = -1; \frac{\partial^2 u}{\partial y^2} \quad finite \quad at \quad y = \pm h = 1 + \phi Cos[2\pi x] \tag{10}$$

Using long wavelength approximation and neglecting the wave number δ, one can reduce Navier Stokes equations:

$$\frac{\partial p}{\partial y}=0 \tag{11}$$

$$\frac{\partial p}{\partial x}=\frac{\partial^2 u}{\partial y^2}-\frac{1}{\gamma^2}\frac{\partial^4 u}{\partial y^4}-G\sin\alpha \tag{12}$$

Solving the Eq.(12) with the boundary conditions (10), we get

$$u=\frac{\partial p}{\partial x}((1+\frac{h^2}{\gamma^2})\frac{y^2}{2}-\frac{h^2}{2}-\frac{1}{\gamma^2}(\frac{y^4}{4}+\frac{h^4}{4}))+\frac{G\sin\alpha}{2}(h^2-y^2)+\frac{(y^2-h^2)}{2\gamma^2}+\frac{1}{\gamma^2}-1 \tag{13}$$

The volumetric flow rate in the wave frame is defined by

$$q=\int_0^h u\,dy=-\frac{\partial p}{\partial x}(\frac{h^3}{3}+\frac{2h^5}{15\gamma^2})+(\frac{h^3}{3}(G\sin\alpha-\frac{1}{\gamma^2})+h(\frac{1}{\gamma^2}-1) \tag{14}$$

The expression for pressure gradient from Eq.(14) is given by

$$\frac{\partial p}{\partial x}=\frac{(\frac{h^3}{3}(G\sin\alpha-\frac{1}{\gamma^2})+h(\frac{1}{\gamma^2}-1)-q}{(\frac{h^3}{3}+\frac{2h^5}{15\gamma^2})} \tag{15}$$

The instantaneous flux Q (x, t) in the laboratory frame is

$$Q(x,t)=\int_0^h (u+1)dy=q+h \tag{16}$$

The average flux over one period of peristaltic wane is $\overline{Q}$

$$\overline{Q}=\frac{1}{T}\int_0^T Q\,dt=q+1 \tag{17}$$

From equations (15) and (17), the pressure gradient is obtained as

$$\frac{\partial p}{\partial x}=\frac{(\frac{h^3}{3}(G\sin\alpha-\frac{1}{\gamma^2})+h(\frac{1}{\gamma^2}-1)-(\overline{Q}-1)}{(\frac{h^3}{3}+\frac{2h^5}{15\gamma^2})} \tag{18}$$

The pressure rise (drop) over one cycle of the wave can be obtained as

$$\Delta P = \int_0^1 \left(\frac{dp}{dx}\right)dx \tag{19}$$

The dimensionless frictional force F at the wall across one wavelength is given by

$$F = \int_0^1 h\left(-\frac{dp}{dx}\right)dx \tag{20}$$

III.RESULT AND DISCUSSIONS

In this section we have presented the graphical results of the solutions axial velocity u, pressure rise ΔP, friction force F for the different values of couple stress (γ), gravitational parameter (G), angle of inclination (α) and amplitude (ϕ). The axial velocity is shown in **Figs. (2 to 5)**.

The Variation of u with γ, we find that u depreciates with increase in γ (**Fig. 2**). The Variation of u with gravitational parameter G shows that for u increases with increasing in G (**Fig 3**). The Variation of u with angle of inclination α shows that for u increases with increasing in α (**Fig 4**). The axial velocity u is exhibit in (**fig. 5**) for a different values of amplitude ϕ in the region y = 0 to y = 1. It is found that the velocity u is increases with increasing ϕ.

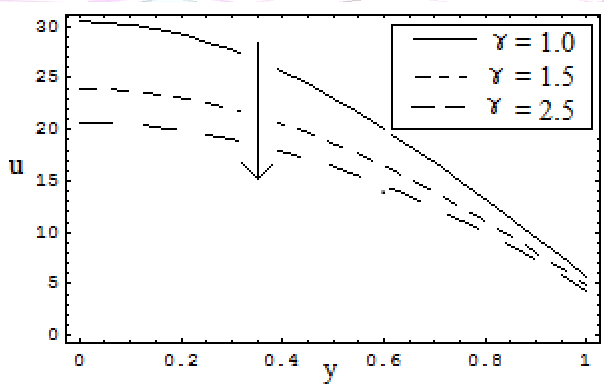

Fig. 2: Effect of γ on u, when ϕ □□□□□□x□□□□□□□□-p5, G = 6 & □ = □/4.

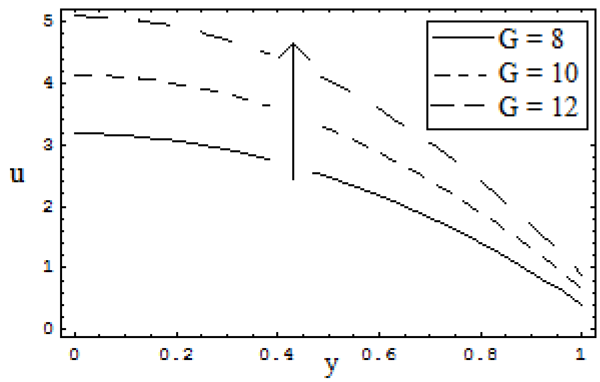

Fig. 3: Effect of G on u when γ ϕ x p = -.25 & = /4

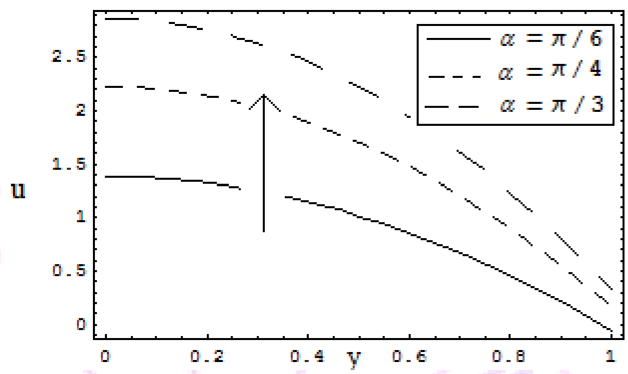

Fig. 4: Effect of α on u when γ ϕ x p = -.25 & G = 6

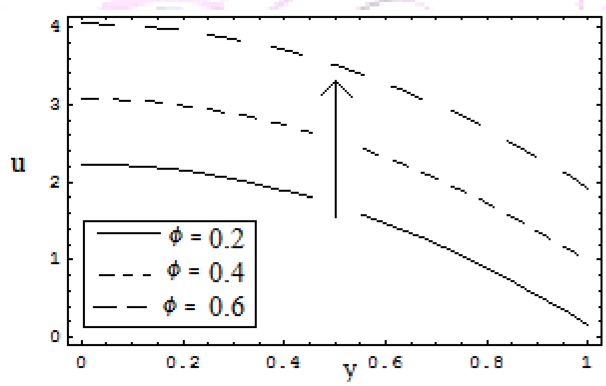

Fig. 5: Effect of ϕ on u when γ K x p = -25, G = 6 & = /4.

The variation of pressure rise ΔP is shown in **Figs. (6 to 9**) for a different values of γ, G, α & ϕ. We find that ΔP depreciates with increase in γ (**Fig. 6**). The Variation of ΔP with gravitational parameter G shows that for ΔP increases with increasing in G (**Fig 7**). The Variation of ΔP with angle of inclination α shows that for ΔP increases with increasing in α (**Fig 8**). The axial velocity ΔP is exhibit in (**fig. 9**) for a different values of amplitude ϕ in the region y = 0 to y = 1. It is found that the velocity ΔP is increases with increasing ϕ.

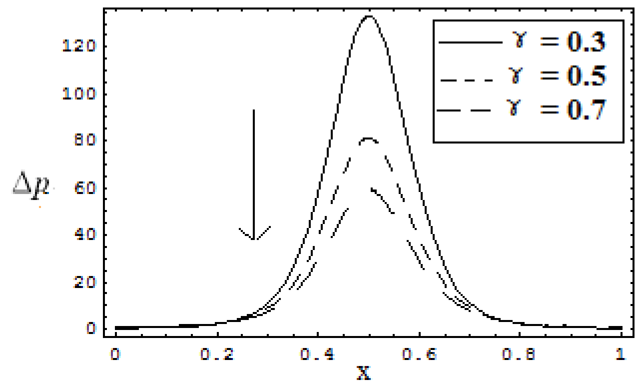

Fig. 6: Effect of γ on Δp when ϕ G = 4, = /4, Q = 0 x

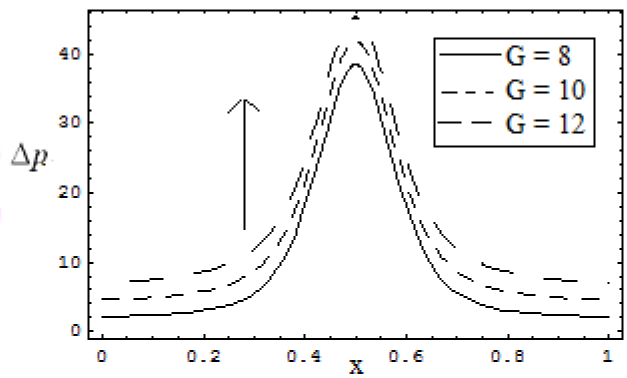

Fig. 7: Effect of G on Δp when γ ϕ x = /4 & Q = 0.

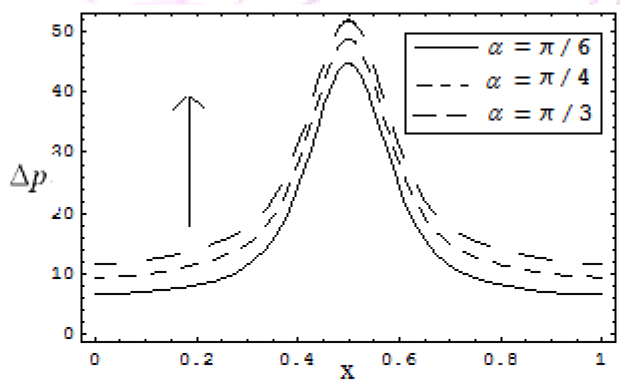

Fig. 8: Effect of α on Δp when γ ϕ x Q=0 G = 20.

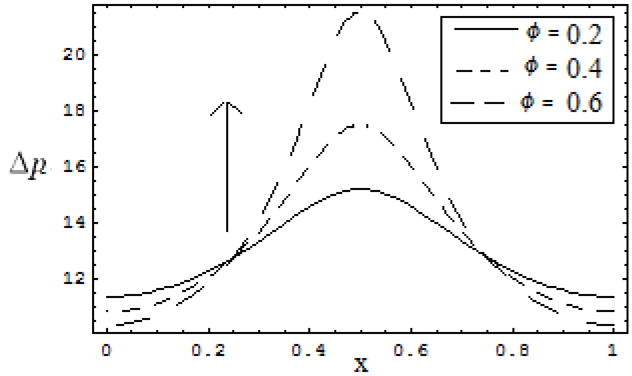

Fig. 9: Effect of ϕ on Δp □ when γ □ □□ □□G = 20, x=0.1, □ = □/4 & Q=0.

The variation of friction force F is shown in **Figs. (10 to 13)** for a different values of γ, G, α & ϕ. We find that F depreciates with increase in γ (**Fig. 10**). The Variation of F with gravitational parameter G shows that for F increases with increasing in G (**Fig. 11**). The Variation of F with angle of inclination α shows that for F increases with increasing in α (**Fig 12**). The friction force F is exhibit in (**fig. 13**) for a different values of amplitude ϕ in the region y = 0 to y = 1. It is found that the F is increases with increasing ϕ.

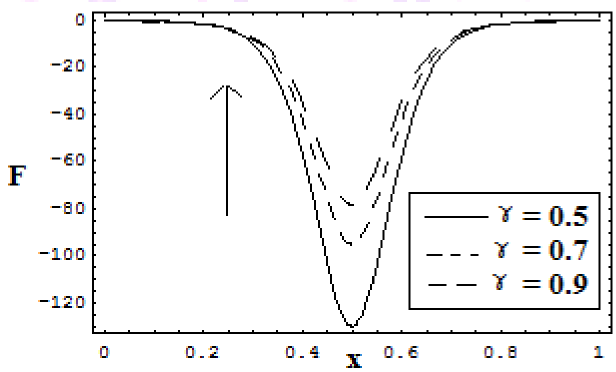

Fig. 10: Effect of γ on F□ whenG = 4□ ϕ □□ □ □□□ = □/4Q = 0 □□ □ □ □ x□ □ □ □ □ □

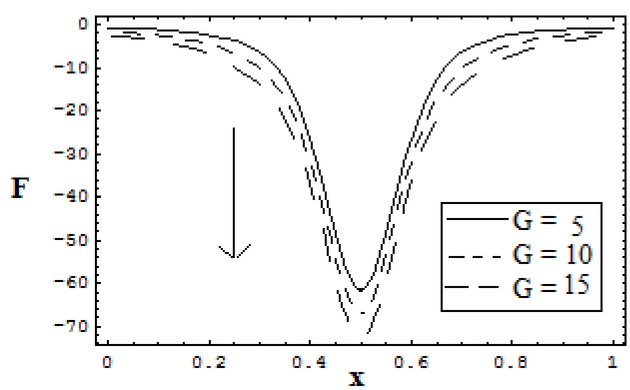

Fig. 11: Effect of G on F□ □ when γ □ □ □ □□ϕ □ □ □ □ □ □□□= □/4, Q = 0□ □ x□ □ □ □ □

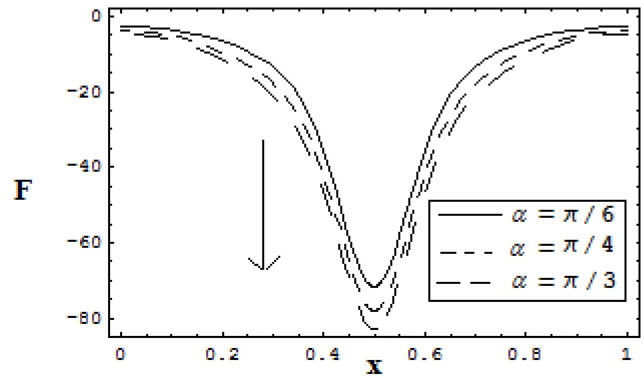

Fig. 12: Effect of α on F when γ ϕ G = 20, Q=0 x

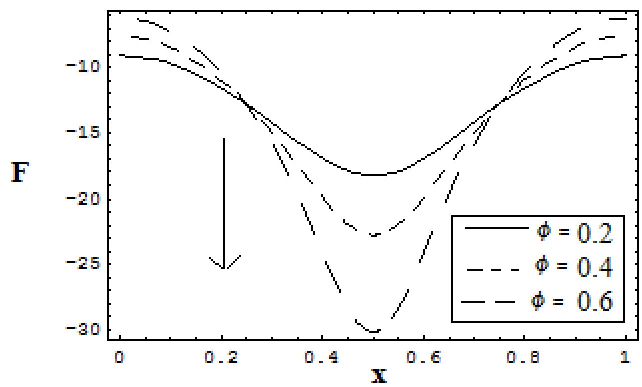

Fig. 13: Effect of ϕ on F when γ G = 20, = /4, Q=0 x

IV. CONCLUSION

In this paper we presented a theoretical approach to study the peristaltic flow of a couple stress fluids in an inclined channel. The governing Equations of motion are solved analytically. Furthermore, the effect of various values of parameters on Velocity, Pressure rise and Friction force have been computed numerically and explained graphically. We conclude the following observations:

1. The velocity u increases with increase in gravitational parameter G, angle of inclination α & amplitude ϕ but, decreases with increase in couple stress parameter γ.

2. Pressure rise ΔP decreases with increase in couple stress parameter γ but, increases with increase in gravitational parameter G, angle of inclination α & amplitude ϕ.

3. The friction force F has increases with increase in gravitational parameter G, angle of inclination α & amplitude ϕ but, decreases with increase in couple stress parameter γ.

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